

QUANTIFIER SEMANTICS—ARISTOTLE'S WAY

PHILIP L. PETERSON

Syracuse University
Syracuse, New York U.S.A.
plpeters@syr.edu

Integrating "few", "many", and "most" into traditional logic produced a syllogistic system of five quantities with 105 valid forms. The algebraic methods for intermediate quantities were extended to fractional, proportional and modified quantifiers. Recent extensions cover the infinite-quantity syllogism. Proportion is the traditional concept of distribution (answering Geach). Extensions to relations have been researched. Empirical observations of English quantifiers confirm the applicability of the new syllogistic (with a potentially infinite number of quantities, indefinite numbers of terms, and complex relational structures). Confirmation in other NLs can be confidently pursued.

KEYWORDS: quantifier; intermediate quantifiers; quantifier semantics; syllogism; distribution; proportion

The core of the semantics for quantifiers concerns what logical roles they play in making assertions and denials, and in expressing inferences and manifesting entailments and presuppositions. The basic idea for how to carry out an *Aristotelean semantics* for quantifiers was introduced (in Peterson *NDJFL* 1979) by considering the linguistic meanings and ordinary uses of the English intermediate quantifier expressions *few*, *many*, and *most*. Discovering how they fit into a traditional square of opposition was the first step—viz.

(A) Most S are P	(E) Most S are not-P
(I) Many S are P	(O) Many S are not-P

ISBN: 0 08 043 438X

while assuming:

(Most S are P) iff (Few S are not-P)

(Most S are not-P) iff (Few S are P)

The crucial presupposition (parallel to Aristotle's on existential import) is the 'presupposition of a constant reference class'. This new square fits into Aristotle's by being intermediate between his universal and particular forms. Also, the phenomenon of 'quantifier negation' (an operation untreated in traditional or contemporary logic) is needed to defend the developments.

The task of completely integrating three such quantifiers into traditional logic was carried out with the help of Robert Carnes (cf. Peterson & Carnes 1981, 1983, and Carnes & Peterson *NDJFL* 1991). The result was a syllogistic system of five quantities (rather than Aristotle's two) wherein there are 4000 argument forms of which 105 are valid. New, but appropriately Aristotelian, rules were developed, rules still crucially dependent on distribution (wherein the Rules of Quantity are still demonstrably dispensable). New methods for representing syllogisms on Venn Diagrams validate all 105 forms. The soundness and completeness of the 5-quantity rules were proved—first following the traditional method of reducing all valid forms to a subset of the traditional 24, and second by emulating contemporary styles for demonstrating soundness and completeness of formal systems.

The algebraic methods for intermediate quantities were extended (in Peterson & Carnes 1981 and Peterson *NDJFL* 1985) to fractional and proportional quantities. When fractional quantities (starting with the contradictories of majority statements) are added to the 5-quantity system, even higher quantity syllogistic systems are obtained. For finite k quantities as high as you like, $s = 32k^3$ and $v = 3k(k + 2)$ where s is the number of syllogistic forms and v is the number of valid forms.

Three things were shown in Peterson 1988. First, it was shown how valid inferences between (non-linguistic) propositions are explanatorily linked to semantic properties of words, phrases, and sentences typically used to express them. Crucial to the analysis is the distinction between sentence meaning and proposition expressible in a use of a sentence. Second, it is explained how the many new inference patterns revealed by recent advances on intermediate quantifiers motivate hypotheses about semantic features of English quantifier expressions. The inference patterns provide new data for grammatical theory, but do not constitute a theory. The resulting semantics of "few", "many", and "most" based on the 5-quantity syllogistic shows that "few" does mean what "not many" does (following McCawley 1981, 14.1) but also shows (*contra* McCawley 1981) that "most" does not on any use or interpretation mean what "not many...not" means (even if they are logically equivalent on one reading of "most"). As a consequence, McCawley's argument (1981, p. 428) for blocking "there" insertion in certain allegedly non-existential sentences fails.

The results on intermediate and fractional quantifiers were further extended to *modified* quantifiers like "*Much more than half*" (Peterson *JPhL* 1991). Some surprising results of the new algebra were: (i) the allegedly vague relation "very much greater than" (">>") can be made precise; and (ii) the strength of ">>" criterion varies *inversely* with *closeness* of "almost"-inferences (i.e., how close to X% of a quantity another quantity must be to be *almost*-X%). Recently, the rules for filtering out valid syllogisms (devised by Carnes in Peterson & Carnes 1981) were revised and extended to cover the infinite-quantity

ISBN: 0 08 043 438X

syllogism—by relaxing the restrictions on "fractional" quantifiers and permitting any rational fraction (between zero and one) to be a quantifier (cf. Peterson *JPhL* 1995*)*. In a *k*-quantity system, the intermediate quantifiers are any distinct ratios (between zero and one) modified with "more than" or "or more". The final system, the *i*-quantity system (to which the revised rules also apply) was produced by combining all possible *k*-quantity systems (where each *k*-quantity system contains a finite number of quantities *k*; e.g., for Aristotle, $k = 2$, for Peterson & Carnes 1981, $k = 5$). After applying these non-algebraic methods to sorities, syllogism-like inferences with quantifiers in the predicates (due to Finch, and earlier Hamilton) were analyzed (cf. Peterson *NDJFL* 1993). The climax of Peterson 1995 was the discovery that the defensible concept of distribution (*vs.* Geach and others) just is the concept of proportion articulated in the *i*-quantity ("iQ") syllogistic.

Extension of the Aristotelean approach to sentences expressing relations (following leads from Sommers and Englebretsen) has been developed by devising a grammatically sensitive extension of the traditional *Dictum di Omni* (cf. Peterson 1995 LMPS-Congress, Peterson 1996 AAAI-Symposium). To add relations to syllogistic systems, the *Dictim di Omni* (DDO) was reformulated first as DDO^P and then DDO^I was developed to cover iQ syllogisms. Finally, DDO* results from extending DDO^I to arguments wherein one or more iQ categoricals is replaced by a (simple or complex) "basic relational categorical" (BRC). Challenges for further research on DDO* include iterations, embedded terms due to n-place relations (n>2), VP-modifiers, and other clausal NPs.

Empirical observations of the uses of English quantifiers (all, each, every, some, none, many, few, most, almost-all, half, one-third, three-fourths, two-hundred-and-thirty-seven-five-hundred-and-twenty-fifths, much more than two-thirds, very much less than one-tenth, etc.) confirms the applicability of the new syllogistic (with a potentially infinite number of quantities, indefinite numbers of "terms", and relational structures as complex as can be expressed). Confirmation in other natural languages (following the methods of Peterson & Wali LA 1985 on facts, propositions, and events) should be pursued.

REFERENCES

Carnes, R. D. and Peterson, P. L. "Intermediate Quantifiers *Versus* Percentages." *Notre Dame Journal of Formal Logic,* 1991, *32*(2), 294-306.

Englebretsen, G. "Linear Diagrams for Syllogistic (with Relationals)." *Notre Dame Journal of Formal Logic*, 1992. *33*(1).

McCawley, J. *Everything that Linguists Have Always Wanted to Know About Logic.* University of Chicago Press, 1981.

Peterson, P. L. "On the Logic of 'Few', 'Many', and 'Most'." *Notre Dame Journal of Formal Logic*, 1979, *20*, 155-179.

_____. "Higher Quantity Syllogisms." *Notre Dame Journal of Formal Logic*, 1985, *26*(4), 348-360.

_____. "Syllogistic Logic and the Grammar of Some English Quantifiers." Indiana University Linguistics Club Publications, May 1988.

_____. "Complexly Fractionated Syllogistic Quantifiers." *Journal of Philosophical Logic*, 1991, *20*, 287-313.

_____. "Intermediate Quantifiers for Finch's Proportions." *Notre Dame Journal of Formal Logic*, 1993, *34*(1), 140-149.

_____. "Distribution and Proportion." *Journal of Philosophical Logic*, 1995, *24*, 193-225.

ISBN: 0 08 043 438X

_____. "Basic Relational Infinite-Quantity Syllogisms." Section 5: Philosophical Logic. 10th Congress of Logic, Methodology, & Philosophy of Science, Florence, Italy. Room A1, Palazzo Degli Affari, 22 August 1995.

_____. "How to Use Aristotelian Logic to Formalize Reasonings Expressed in Ordinary Language." AAAI-96 Symposium on *Knowledge Representations Systems Based on Natural Language*, Massachusetts Institute of Technology, November 9-11, 1996 (*Proceedings*, pp. 106-119).

Peterson, P. L. and Carnes, R. D. "The Compleat Syllogistic: Including Rules and Venn Diagrams for Five Quantities. Ms., Syracuse University, 1981(-1985).

_____. "Syllogistic Systems with Five or More Quantities." Read to the Annual Meetings of the Association for Symbolic Logic, Denver, 7 January 1983. (Abstract in *Jour. Sym. Logic* 1984.)

Peterson, P. L. and Wali, Kashi. "Event." *Linguistic Anlysis,* 1985, *15*(1), 3-18.

Sommers, F. *The Logic of Natural Language.* Oxford: Clarendon Press, 1982.

ISBN: 0 08 043 438X